



Analysis of a SIMO transceiver system for Rayleigh fading environment with maximal ratio combining

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Abstract - To achieve the desired goal of increasing the SNR and reducing BER in wireless communication, it is required to mitigate fading of signal during transmission. If more than one antenna is used for transmission and reception, it will improve the performance. The reliability of the transmission of the wireless communication can be enhanced by minimizing the channel fluctuation due to fading. This minimization process can be achieved by different diversity techniques. The central idea in diversity is that different receiving antenna will receive different version of the same signal, which will increase the chances of small deep fading of signal. In this paper we have analyzed the performance of a multiple receiver antenna system using maximal ratio combining diversity technique for different parameters like BER, SNR.

Key words: SIMO, MRC, Rayleigh fading, Diversity

I. INTRODUCTION

Different type of technique is used for different type of impairments mitigation[2]. Diversity is used for reducing flat fading, equalization is used to overcome ISI, and error correction coding is used for identifying error and for correction. The main goal is to transmit a signal so that at the receiver side residual errors are as low as possible. A successful method to improve reliable communication over a wireless link is to use multiple antennas. The average increase in signal power is proportional to the number of receive antennas. Co-channel interference contributes to the overall noise of the system and deteriorates performance[4].

By using multiple antennas it is possible to suppress interfering signals what leads to an improvement of system capacity[7]. An effective method to reduce fading is diversity. According to the domain where diversity is introduced, diversity techniques are classified into time, frequency and space diversity. Space diversity is popular and mostly used in wireless microwave communications and can be classified into two categories: receive diversity and transmit diversity, depending on whether multiple antennas are used for reception or transmission. Receiver diversity can be used in channels with multiple antennas at the receive side. The receive signals are assumed to fade independently and are combined at the receiver so that the resulting signal shows significantly reduced fading. In general the system performance with diversity techniques depends on how many signal replicas are combined at the receiver to increase the overall SNR. There exist four main types of signal combining methods at the receiver: selection combining, switched combining, equal-gain combining and maximum ratio combining (MRC). In this paper we used the Maximal ratio combining technique. We have used MATLAB to simulate the performance of wireless communication for receiver diversity using maximal ratio combining [6].

II. MODELLING OF MIMO SYSTEM

A wireless communication system consists of a transmitter, a channel and a receiver. This simplest configuration uses single antenna at both sides and is denoted as single input/single output system (SISO). If multiple antenna is used at one or both sides of the channel are denoted as multiple input and multiple output (MIMO) system. Systems with multiple antennas on the receive side only are called single input/multiple output (SIMO) systems and systems with multiple antennas at the transmitter side and a single antenna at the receiver side are called multiple input/single output (MISO) systems. The MIMO system is the most general and includes SISO, MISO, and SIMO systems as special cases. Therefore, the term MIMO is used in general for multiple antenna systems [5].

For a MIMO system where n_t, n_r are transmit and receive antennas respectively and $h_{p,k}$ is a complex gain corresponding to the channel gain between transmit antenna k and receive antenna p . Such a system is represented in Figure 1. If $\{s_1, s_2, s_3, \dots, s_{n_t}\}$ are the complex signals which are transmitted via n_t transmit antennas, the received signals at antenna can be expressed by [8] equation 2.1

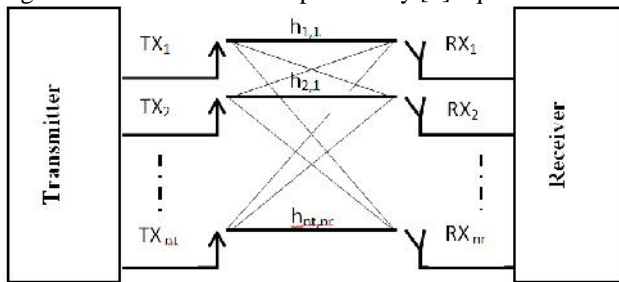


Figure 1: MIMO model with n_t transmitting antennas and n_r receiving antennas

$$y^p = \sum_{k=1}^{n_t} h_{p,k} s_k + n^t \quad (2.1)$$

Combining all receive signals (2.1) can be easily expressed in matrix form in a vector y

$$y = Hs + n \quad (2.2)$$

y is the $n_r \times 1$ receive symbol vector, H is the $n_r \times n_t$ MIMO channel transfer matrix,

$$H = \begin{bmatrix} h_{11} & \dots & h_{1n_t} \\ \vdots & \ddots & \vdots \\ h_{n_r 1} & \dots & h_{n_r n_t} \end{bmatrix} \quad (2.3)$$

s is the $n_t \times 1$ transmit symbol vector and n is the $n_r \times 1$ additive noise vector. Note that the system model implicitly assumes a flat fading MIMO channel, i.e., channel coefficients are constant during the transmission of several symbols.

III. DIVERSITY RECEPTION WITH MAXIMAL RATIO COMBINING

Let, the transmission of digital modulating signal $x(t)$ over a slow rayleigh fading channels using coherent demodulation with L -th order diversity. The received signal component of l -th diversity is [1, p126-130],

$$r_l(t) = \alpha_l(t) \exp(j\theta_l(t)) x(t) + w_l(t) \quad l = 1, 2, \dots, L \quad (3.1)$$

Here where $\alpha(t)$ is the amplitude fading and $\theta(t)$ is the phase distortion and $w(t)$ is the Gaussian noise with zero mean and two-sided power spectral density $N_0/2$.

The output of the demodulator of the l -th diversity at the of the k -th symbol interval $\alpha_{lk} \exp(j\theta_{lk}) + n_{lk}$ where, $\alpha_{lk} \exp(j\theta_{lk}) =$ Complex channel gain of l th channel over the k -th symbol interval. $n_{lk} =$ Vector representation of the transmitted signal over k th symbol interval in the N -Th dimensional signal space and also demodulator output over AWGN channel

$n_{lk} = (n_{lk,1}, n_{lk,2}, \dots, n_{lk,N}) =$ Vector representation of the noise component of the demodulator output due to $w_l(t)$.

Let us consider the maximal ratio combining, as shown in the Figure 2 [1,p127].

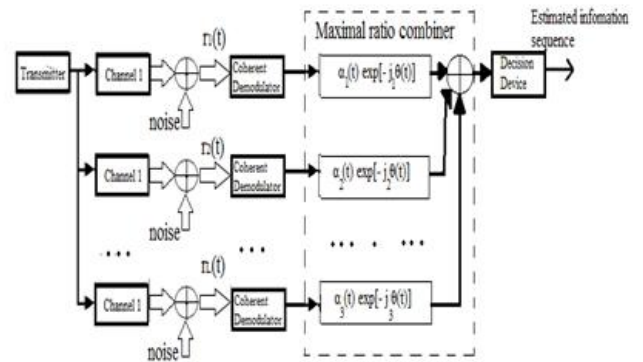


Figure 2: Diversity reception with maximal ratio combining [1, p127]

The decision variable for the k -th transmitted can be represented as,

$$\begin{aligned} r_{lk} &= \sum_{l=1}^L \alpha_{lk} \exp(-j\theta_{lk}) [\alpha_{lk} \exp(j\theta_{lk}) x_k + n_{lk}] \\ &= \left[\sum_{l=1}^L \alpha_{lk}^2 \right] x_k + \left[\sum_{l=1}^L \alpha_{lk} \exp(-j\theta_{lk}) n_{lk} \right] \\ &= g_k x_k + n_k \end{aligned}$$

Where,

$$g_k = \sum_{l=1}^L \alpha_{lk}^2$$

$$\vec{g} = \sum_{l=1}^L \alpha_{lk} \exp(-j\theta_{lk})$$

For the noise vector $\vec{n}_k = (n_{k,1}, n_{k,2}, \dots, n_{k,N})$ each component is,

$$n_{k,l} = \sum_{i=1}^N \alpha_{lk} \exp(-j\theta_{lk}) n_{lk,N}, \quad l = 1, 2, \dots, N$$

Therefore given the weighting vector gain $\alpha_{lk} \exp(-j\theta_{lk})$, $l = 1, 2, \dots, L$, each noise component $n_{k,n}$ is a Gaussian random variable with zero mean and variance,

$$\sigma_{k,n}^2 = \frac{N_0}{2} \sum_{l=1}^L \alpha_{lk}^2$$

For BPSK, $N=1$. The vectors $\vec{r}_k, \vec{x}_k, \vec{n}_k$ can be represented by the corresponding scalar variables r_k, x_k, n_k respectively. The decision variable for k -th transmitted symbol is,

$$r_k = g_k x_k + n_k$$

Where, $x_k = \sqrt{E_b}$ for symbol "1" and $x_k = -\sqrt{E_b}$ for symbol "0". The SNR per bit at the output of the k -th symbol is then,

$$\gamma_k = \frac{[g_k x_k]^2}{2\sigma_{k,n}^2} = \frac{[\sum_{l=1}^L \alpha_{lk}^2 x_k]^2}{2N_0 \sum_{l=1}^L \alpha_{lk}^2} = \frac{E_b}{N_0} \sum_{l=1}^L \alpha_{lk}^2 \quad (3.2)$$

Where $\frac{E_b}{N_0}$ is the SNR value for the AWGN channel with $\alpha_{lk}=1$ and $L=1$.

In a Rayleigh fading environment, the α_{lk} 's are the rayleigh random variables with parameter σ_{α}^2 . Therefore γ_k follows a chi-square distribution with $2L$ degrees of freedom. Its pdf is given by,

$$f_{\gamma}(x) = \frac{x^{L-1} \exp(-x/\Gamma_c)}{(L-1)! \Gamma_c^L}, \quad x \geq 0 \quad (3.3)$$

Where $\Gamma_c = 2 \sigma_{\alpha}^2 E_b / N_0$ is the average SNR per bit in each diversity channel. The mean SNR per Bit after the combining is

$$\Gamma_b = \Gamma_c = 2 \sigma_{\alpha}^2 E_b / N_0 \quad (3.4)$$

The analysis of the probability of error over a Rayleigh fading channel can be extended to case of diversity as,

$$P_b = \int_0^{\infty} P_{e|\gamma}(x) f_{\gamma}(x) dx \quad (3.5)$$

Where $P_{e|\gamma}(x)$ is the conditional probability of bit error, given that the received SNR per bit is x . For coherent BPSK, we have,

$$P_{e|\gamma}(x) = \frac{Q(\sqrt{2x})}{2} \quad (3.6)$$

Substituting equation (3.6) into equation (3.5) we get,

$$P_b = [0.5(1 - \mu)]^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \frac{5(1+\mu)^L}{[0.5(1+\mu)]^L} \quad (3.7)$$

Where,

$$\mu = \sqrt{\frac{\Gamma_c}{1 + \Gamma_c}} \quad (3.8)$$

For $\Gamma_c \gg 1$, we have, $0.5(1+\mu) \approx 1$ and $0.5(1-\mu) \approx 1/4 \Gamma_c$.

Furthermore,

$$\sum_{l=0}^{L-1} \binom{L-1+l}{l} = (2^L - 1) \quad (3.9)$$

As a result, for $\Gamma_c \gg 1$ the probability of error can be approximated as,

$$P_b \approx \left(\frac{1}{4\Gamma_c}\right)^L (2^L - 1) \quad (3.10)$$

Equation (3.6) or (3.8) is obtained where no of transmitting antenna is 1 and the number is receiving antennas are L . So the system is SIMO in Rayleigh fading environment over uncorrelated channel where each transmitting antenna transmits digital signals at the same time and in the same bandwidth using the same symbol period.

Maximal Ratio Combining can be used to find out probability of bit error for MIMO system. If there are N transmitting antenna and L receiving antenna, signal from the first transmitting antenna will go to L receiving antenna. So the diversity channel will be L for the first antenna. The probability of bit error for after combining L diversity channel at the receiving side will follow equation (3.6)

$$P_{b(N,L)} = [0.5(1 - \mu)]^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \frac{5(1+\mu)^L}{[0.5(1+\mu)]^L} \quad (3.11)$$

For the other transmitting antenna same thing will happen. So the probability of bit error for N transmitting antenna will be

$$P_{b(N,L)} = \left\{ [0.5(1 - \mu)]^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \frac{5(1+\mu)^L}{[0.5(1+\mu)]^L} \right\}^N \quad (3.12)$$

Or from equation (3.10)

$$P_{b(N,L)} \approx \left\{ \left(\frac{1}{4\Gamma_c}\right)^L (2^L - 1) \right\}^N$$

IV. SIMULATION AND RESULT

A. BER Analysis without Fading (for SIMO)

We have described before that received SNR per bit after combining L diversity channel is Γ_b , and here we assumed the value of Γ_b is 5dB, 10dB, 15dB, 20dB, 25dB, 30dB, 35dB, 40dB and 45dB gradually. Then using the equation

(3.6) ($\Gamma_b = L\Gamma_c$), Γ_c has been calculated. And from Γ_c and using equation (3.8) μ is obtained. Finally using equation (3.7) BER has been calculated and by using MATLAB we have got the Figure 3

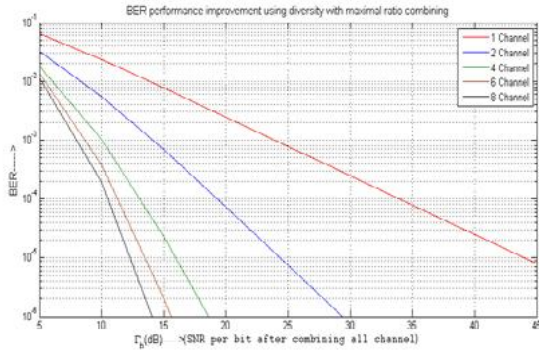


Figure 3: BER performance improvement using diversity with maximal ratio combining

Using equation (3.7) we can observe the SNR improvement. Equation can be reduced to equation (3.10) (assuming $\Gamma_c \gg 1$)

$$P_b \approx \left(\frac{1}{4\Gamma_c}\right)^L \binom{2L-1}{L}$$

From this equation SNR improve has been analyzed taking $BER = 10^{-6}$ we have the following Figure4.

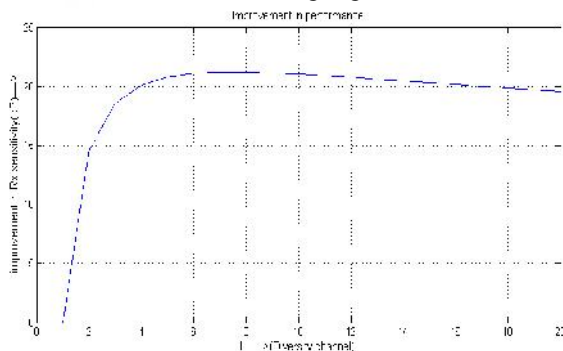


Figure 4: Improvement of SNR using diversity with maximal ratio combining

From Figure 3,4 we can see that SNR improvement is the highest while increasing the diversity channel from 1 to 2. SNR improvement for increasing the diversity channel from 2 to 3 is lower than the SNR improvement from 1 to 2 and from 3 to 4 improvement is lower than the SNR improvement from 2 to 3 and so on. After a specific value of diversity SNR does not improve. Rather it decreases. So we observed that increasing diversity channel improve the SNR for an optimum value of diversity channel. Increasing of diversity channel beyond the optimum value, SNR improvement is negative.

In our analysis for a SIMO system in Rayleigh fading environment when the BER is 10^{-4} we have found that Improvement of SNR for increasing the diversity channel from 1 to 2 is approximately 14.6 dB; Improvement of SNR for increasing the diversity channel from 2 to 3 is

approximately 3.96 dB and Improvement of SNR for increasing the diversity channel from 3 to 4 is approximately 1.6 dB and so on. If we further increase diversity channel, SNR decreases. So here the number of optimum diversity channel is 7.

B. Improvement of with different fading (for SIMO)

Equation (3.10) is given below (where $\Gamma_c \gg 1$):

$$P_b \approx \left(\frac{1}{4\Gamma_c}\right)^L \binom{2L-1}{L}$$

Here we assumed value the of Γ_b is 12000 dB. Using the equation $\Gamma_b = L\Gamma_c$, we have calculated Γ_c . Then using the equation (3.8), SNR improvement with different value of fading variance such us 0.1, 0.05, 0.01, 0.001 has been observed and we have got the following Figure 5.

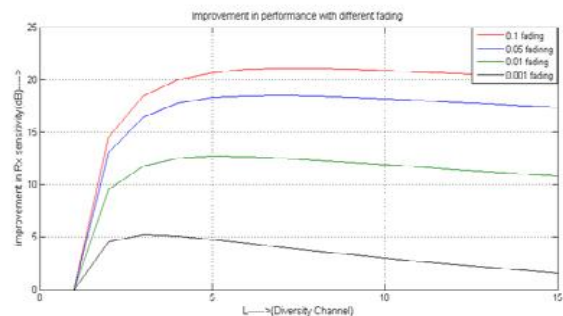


Figure 5: Improvement of SNR for different fading using diversity with maximal ratio combining

From the Figure5 we can see that for higher value of fading, improvement of SNR is better for a Rayleigh fading environment. Where the fading is low SNR improvement is not significant. For the specific value used here, we have got that if we increase diversity channel from 1 to 2, following will happen:

- a) Improvement of SNR is approximately 15.82 dB when fading is 0.1.
- b) Improvement of SNR is approximately 13.01 dB when fading is 0.05.
- c) Improvement of SNR is approximately 9.51dB when fading is 0.01.
- d) Improvement of SNR is approximately 4.58 dB when fading is 0.001.

If we increase diversity channel from 2 to 3, following will happen:

- a) Improvement of SNR is approximately 3.92 dB when fading is 0.1.
- b) Improvement of SNR is approximately 3.43 dB when fading is 0.05.
- c) Improvement of SNR is approximately 2.18 dB when fading is 0.01.

d) Improvement of SNR is approximately 0.61 dB when fading is 0.001.

So the interesting point is that decreasing fading value decreases the SNR improvement.

V. CONCLUSION

From our study we have found that for wireless communication in order to get good performance with better SNR and BER we have to use multiple antennas at transmitter and receiver side[3]. In this paper we have only analyzed the performance for multiple antennas at receiving which results better performance. We have analyzed the performance of maximal ratio combining(MRC) as diversity technique for mitigating the decrease SNR for Rayleigh fading.

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